CRC Retreat 2023: Abstracts

4 – 6 September 2023

Integrable systems and Lie algebra decompositions

Stepan Maximov

Until the middle of the 20th century it was a common belief, that complicated non-linear physical systems exhibit an ergodic behaviour. Everything changed with the FPUT experiment, which showed that a certain system of N oscillators with non-linear interactions behaves periodically. Searching for a rigorous explanation of the observed phenomena leads to many interesting notions, including Lax pairs, Lie bialgebras, Lie dialgebras, (generalised) classical Yang-Baxter equation etc. At the heart of these notions, as well as project A5, lies the idea of a Lie algebra decomposition. This talk is about the current project on regular decompositions of certain Lie algebras, which can be used for the construction of new explicit integrable systems.

Theta lifts and equidistribution

Claudia Alfes-Neumann and Rebekka Strathausen

In this talk we will first give an introduction to theta liftings and their role in the theory of automorphic forms. We will review equidistribution results which can be proved via considering certain theta liftings (CAN). More concretely, we will present a twisted version of the so-called Katok-Sarnak formula and explain how this might lead to a new viewpoint on the mixing conjecture of Michel and Venkatesh (RS).

A symplectic view of matrix factorizations

Kyungmin Rho

Recently, homological mirror symmetry has revealed strong connections between representation theory and symplectic geometry. We will show one such application on matrix factorizations that have been classically important in mathematics and physics. In terms of representation theory, Burban-Drozd (2017) classified all indecomposable matrix factorizations of xyz. In view of symplectic geometry, we find the corresponding curves in a surface under the mirror symmetry. We use them to establish the canonical form of matrix factorizations of xyz. This is based on joint works with Cheol-Hyun Cho, Wonbo Jeong, and Kyoungmo Kim.

The hat tiling is topologically conjugate to a model set

Franz Gähler

The by now well-known hat tiling is related by a shape change to a self-similar (meta-tile) tiling with geometric ϕ^2 inflation. We show that this shape change is asymptotically negligible in the sense of Clark/Sadun, meaning that it does not mess up the long-range aperiodic order. As a result, these tilings form conjugate dynamical systems under the translation action, and their dynamical spectra must coincide. By the overlap algorithm, we can show that the spectra of both tilings are pure-point.

Inflation tilings with pure-point spectrum are expected to be cut-and-project sets (model sets), which is also the case here. We construct an equivalent cut-and-project set obtained from a $4d$ lattice with triangular symmetry and

ϕ^2 scaling, and a reasonably simple window having a regular hexagon as outer shape. Only the internal boundaries between different tile types are fractal.

We also briefly mention that the properties of the spectre tiling are completely analogous to those of the hat tiling.

On the asymptotics of elementary-abelian extensions over global function fields

Nicolas Potthast

In 1989, David Wright determined the asymptotic behaviour of abelian extensions of global fields, counted by discriminant. Even though Wright restricted his considerations to Galois groups of order coprime to the characteristic, he suggested analogous results for the general case. Further research by Thorsten Lagemann, however, who studied Wright's omitted case for local function fields, disproved Wright's suggestion and showed that this case is indeed more complicated. In this talk, we focus on the omitted case as well and count Galois extensions with elementary-abelian group C_p^r of global function fields of characteristic p by their discriminant. We determine the number of these extensions for a given fixed discriminant and study the Dirichlet series of the corresponding arithmetic function. Since an arbitrary global function field as base field involves further technical difficulties like non-trivial class groups and Selmer groups, we first restrict ourselves to the field of rational functions over a finite field. In this case, we can precisely decompose the Dirichlet series into a linear combination of Euler products and we determine the asymptotics of those extensions by analysing these Euler products. Afterwards, we describe the modifications in the arithmetic function of an arbitrary base field compared to the case of the field of rational functions.

On weakly chordal graphic arrangements

Leonie Mühlherr

A hyperplane arrangement is called free if its module of derivations is free. A graphic arrangement is a subarrangement of the braid arrangement whose set of hyperplanes is determined by an undirected graph. A classical result due to Stanley, Edelman and Reiner states that a graphic arrangement is free if and only if the corresponding graph is chordal, i.e., the graph has no chordless cycle with four or more vertices. In this talk, we first present the concept of freeness in the graphic setting and extend it to the case of graphic arrangements of projective dimension at most 1, whose underlying graphs form the class of weakly chordal graphs (a graph is weakly chordal if the graph and its complement have no chordless cycle with five or more vertices). This is joint work with Takuro Abe, Lukas Kühne and Paul Mücksch.

Spectral theory of locally symmetric spaces

Lasse Wolf

In this talk I will give an overview of Project B2. We will review the spectral theory of tori as well as the modular surface. This motivates the study of general locally symmetric spaces especially quotients by thin groups. We will recall the theorems on their spectra by Patterson and mention first results in higher rank.

Reflection groups and Artin groups: classical and elliptic theories

Georges Neaime

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